Limits, Continuity and Differentiability

Question1

$\lim_{x\to1} \frac{x^4-\sqrt{x}}{\sqrt{x}-1}$ **is**

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Options:

A. 0

B. 7

C. Does not exist

D. $\frac{1}{2}$

Answer: B

Solution:

$$\lim_{x\to1} \frac{x^4-\sqrt{x}}{\sqrt{x}-1} = \lim_{x\to1} \frac{4x^3-\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to1} \frac{\frac{8x^3\sqrt{x}-1}{2\sqrt{4}}}{\frac{1}{2\sqrt{x}}}$$
$$= 7$$

Question2

Match the following:

In the following, $[\mathrm{x}]$ denotes the greatest integer less than or equal to x .

Column - I		Column - II	
(a)	$x\lvert x\rvert$	(i)	continuous in (-1, 1)
(b)	$\sqrt{\lvert x\rvert}$	(ii)	differentiable in (-1, 1)

(c)	x + [x]	(iii)	strictly increasing in (-1, 1)
(d)	|x − 1| + |x + 1|	(iv)	not differentiable at, at least one point in (-1, 1)

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Options:

A. $\mathrm{a-i, b-ii, c-iv, d-iii}$

B. $a-iv, b-iii, c-i, d-ii$

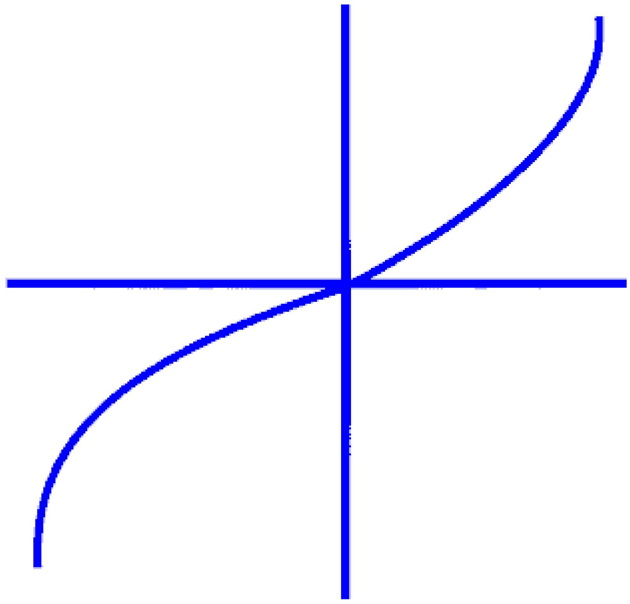
C. a - ii, b - iv, c - iii, d - i

D. a - iii, b - ii, c - iv, d - i

Answer: C

Solution:

(a) $x|x|$ $f(x)=\begin{cases} x^2, & x \geq 0 \\ -x^2 & x<0 \end{cases}$ differentiable in $(-1,1)$



(b) $\sqrt{|x|}=\begin{cases} \sqrt{x}, & x \geq 0 \\ \sqrt{-x}, & x<0 \end{cases}$ Not differentiable at $x=0$

(c) x + [x] strictly increasing in $(-1,1)$

(d) $|x-1|+|x+1|=\left\{\begin{array}{cc}-x+1-x-1 & x<-1 \ -x+1+x+1 & -11\end{array}\right.$

$$\text{Continuous } (-1,1) = \begin{cases} -2x & , x < -1 \\ 2, & -1 < x < 1 \\ 2x & , x > 1 \end{cases}$$

Question3

The function $f(x) = \begin{cases} e^x + ax & , x < 0 \\ b(x-1)^2 & , x \geq 0 \end{cases}$ is differentiable at $x = 0$. Then

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Options:

A. $a = 1, b = 1$

B. $a = 3, b = 1$

C. $a = -3, b = 1$

D. $a = 3, b = -1$

Answer: C

Solution:

$$f(x) = \begin{cases} e^x + ax & , \quad x < 0 \\ b(x-1)^2 & , \quad x \geq 0 \end{cases}$$

Continuity LHL $= 1$

RHL $= b \Rightarrow b = 1$

Differentiability LHD $= 1 + a$

RHD $= -2b$

$1 + a = -2b$

$a = -3$

Question4

A function $f(x)=\begin{cases}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$

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Options:

A. continuous at x $=0$

B. not continuous at $x=0$

C. differentiable at $x=0$

D. differentiable at x $=0$, but not continuous at x $=0$

Answer: B

Solution:

$$f(x)=\begin{cases}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & , \quad x\neq 0\\ 0 & , \quad x=0\end{cases}$$

LHL $=-1$, RHL $=1$, Not continuous.

Question5

$\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\cos x-1}{\cot x-1}$ **is equal to**

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Options:

A. 2

B. $\sqrt{2}$

C. $1/2$

D. $1/\sqrt{2}$

Answer: C

Solution:

$\because \lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$

$$= \lim_{x \to \frac{\pi}{4}} \frac{-\sqrt{2}\sin x}{-\operatorname{cosec}^2 x} \text{ [using L-Hospital rule]}$$
$$= \frac{-\sqrt{2} \times \frac{1}{\sqrt{2}}}{-2} = \frac{1}{2}$$

Question6

Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$. Then, $\lim_{x \to 0} \frac{f(x)}{x^2}$ is

KCET 2024

Options:

A. -1

B. 0

C. 3

D. 2

Answer: B

Solution:

$$\because f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$$
$$= \cos x \left(x^2 - 2x^2\right) - x(2x\sin x - 2x\sin x)$$
$$= -x^2\cos x + x\sin x$$

Now,

$$\lim_{x\to 0}\frac{f(x)}{x^2}=\lim_{x\to 0}\left(\frac{-x^2\cos x+x\sin x}{x^2}\right)$$
$$=\lim_{x\to 0}\left(-\cos x+\frac{\sin x}{x}\right)$$
$$=-1+1\quad\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$
$$=0$$

Question7

The function $f(x)=|\cos x|$ is

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Options:

A. Everywhere continuous and differentiable

B. Everywhere continuous but not differentiable at odd multiples of $\pi/2$

C. Neither continuous nor differentiable at $(2n+1)\frac{\pi}{2}, n\in Z$

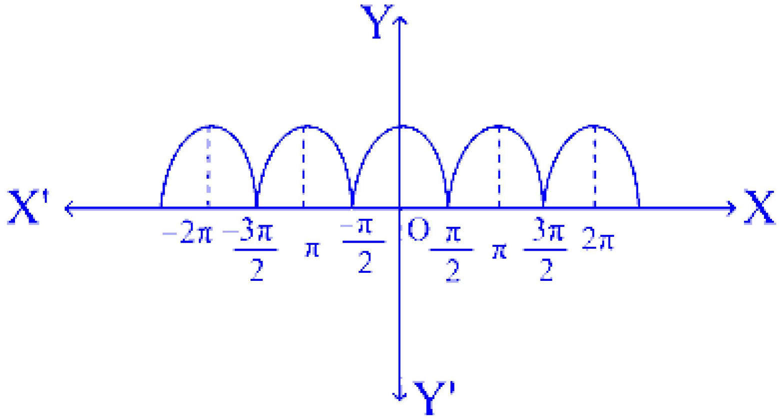
D. Not differentiable everywhere

Answer: B

Solution:

$f(x)=|\cos x|$

Graph of $f(x)=|\cos x|$



From graph we can see that $f(x)$ is not differentiable at points $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{-\pi}{2}$ . . .. . Also, it is everywhere continuous.

i.e. At $x = (2n+1)\frac{\pi}{2}, n \in Z$

$f(x)$ is not differentiable but continuous everywhere.

Question8

$$\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots+\frac{1}{5n}\right)=$$

KCET 2024

Options:

A. $\pi/4$

B. $\tan^{-1} 3$

C. $\tan^{-1} 2$

D. $\pi/2$

Answer: C

Solution:

$$\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots+\frac{1}{5n}\right)$$

$$=\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots+\frac{n}{n^2+(2n)^2}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2}$$

$$=\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{1}{n}\left(\frac{1}{1+\left(\frac{r}{n}\right)^2}\right)$$

$$=\int_0^2\frac{dx}{1+x^2}=\tan^{-1}(2)$$

Question9

If $\lim_{x \to 0} \frac{\sin(2+x) - \sin(2-x)}{x} = A \cos B$**, then the values of** A **and** B **respectively are**

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Options:

A. $1, 2$

B. $2, 1$

C. $1, 1$

D. $2, 2$

Answer: D

Solution:

We have, $\lim_{x \to 0} \frac{\sin(2+x) - \sin(2-x)}{x} = A \cos B$

$\lim_{x \to 0} \frac{2 \cos\left(\frac{2+x+2-x}{2}\right) \sin\left(\frac{2+x-2+x}{2}\right)}{x}$

$= \lim_{x \to 0} 2 \cos 2 \frac{\sin x}{x}$

$= 2 \cos 2 \lim_{x \to 0} \frac{\sin x}{x} = 2 \cos 2$

Thus, $A = 2$ and $B = 2$

Question10

The function $f(x) = \cot x$ **is discontinuous on every point of the set**

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Options:

A. $\{x = 2n\pi; n \in Z\}$

B. $\{x = (2n+1)\frac{\pi}{2}; n \in Z\}$

C. $\{x = \frac{n\pi}{2}; n \in Z\}$

D. $\{x = n\pi; n \in Z\}$

Answer: D

Solution:

Given, $f(x) = \cot x$

$\Rightarrow \quad f(x) = \frac{\cos x}{\sin x}$

We know that $\sin x = 0$, if $f(x)$ is discontinuous

$\therefore$ If $\sin x = 0$

$\therefore x = n\pi, n \in n\pi$

So, the given function $f(x)$ is discontinuous on the set $\{x = n\pi, n \in Z\}$

Question11

If $f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \leq x < 3 \end{cases}$**'**

the quadratic equation whose roots are $\lim_{x \to 2^-} f(x)$ **and** $\lim_{x \to 2^+} f(x)$ **is**

KCET 2022

Options:

A. $x^2 - 14x + 49 = 0$

B. $x^2 - 10x + 21 = 0$

C. $x^2 - 6x + 9 = 0$

D. $x^2 - 7x + 8 = 0$

Answer: B

Solution:

$$f(x)=\begin{cases}x^2-1, & 0<x<2\\ 2x+3, & 2\le x<3\end{cases}$$

Now, $\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^-} x^2-1=\lim\limits_{h\to 0}\left[(2-h)^2-1\right]=\lim\limits_{h\to 0}\left[4+h^2-4h-1\right]=4+0-0-1=3$

and $\lim\limits_{x\to 2^+} f(x)=\lim\limits_{x\to 2^+}(2x+3)=\lim\limits_{h\to 0}2(2+h)+3$
$=\lim\limits_{h\to 0}4+2h+3=4+0+3=7$

Therefore, the quadratic equation whose roots are 3 and 7 is given by $x^2-(3+7)x+(3\times 7)=0$ which is $x^2-10x+21=0$

Question12

$$\lim_{y\to 0}\frac{\sqrt{3+y^3}-\sqrt{3}}{y^3}=$$

KCET 2022

Options:

A. $\frac{1}{2\sqrt{3}}$

B. $\frac{1}{3\sqrt{2}}$

C. $2\sqrt{3}$

D. $3\sqrt{2}$

Answer: A

Solution:

$$\text{Let } L = \lim_{y \to 0} \frac{\sqrt{3+y^3}-\sqrt{3}}{y^3}$$

$$L = \lim_{y \to 0} \frac{\left(\sqrt{3+y^3}-\sqrt{3}\right)}{y^3} \times \frac{\left(\sqrt{3+y^3}+\sqrt{3}\right)}{\left(\sqrt{3+y^3}+\sqrt{3}\right)}$$

$$= \lim_{y \to 0} \frac{3+y^3-3}{y^3\left(\sqrt{3+y^3}+\sqrt{3}\right)} = \lim_{y \to 0} \frac{y^3}{y^3\left(\sqrt{3+y^3}+\sqrt{3}\right)}$$

$$= \lim_{y \to 0} \frac{1}{\sqrt{3+y^3}+\sqrt{3}} = \frac{1}{\sqrt{3}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

Question13

Consider the following statements

Statement 1 : $\lim\limits_{x \to 1} \frac{ax^2+bx+c}{x^2+bx+a}$ **is 1**

(where $a+b+c \neq 0$**).**

Statement 2 : $\lim\limits_{x \to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2}$ **is** $\frac{1}{4}$**.**

KCET 2021

Options:

A. Only statement 2 is true.

B. Only statement 1 is true.

C. Both statements 1 and 2 are true.

D. Both statements 1 and 2 are false.

Answer: B

Solution:

Explanation for Statement 1:

Given the limit:

$\lim_{x \to 1} \frac{ax^2+bx+c}{x^2+bx+a}$

Evaluating this limit at $x = 1$ gives:

$\frac{a \times 1^2 + b \times 1 + c}{1^2 + b \times 1 + a} = \frac{a+b+c}{a+b+c} = 1$

As long as $a + b + c \neq 0$, dividing the numerator and the denominator by the same non-zero quantity results in 1. Therefore, Statement 1 is true.

Explanation for Statement 2:

Consider the limit:

$\lim_{x \to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2}$

This can be rewritten as:

$\lim_{x \to -2} \frac{\frac{2+x}{2x}}{x+2}$

Simplifying further, we have:

$\lim_{x \to -2} \frac{1}{2x}$

Substituting $x = -2$, we find:

$\frac{1}{2 \times (-2)} = -\frac{1}{4}$

Thus, the correct limit is $-\frac{1}{4}$, not $\frac{1}{4}$ as stated. Therefore, Statement 2 is false.

Question14

If $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 0 & 2\cos x & 3 \\ 0 & 1 & 2\cos x \end{vmatrix}$**, then** $\lim_{x \to \pi} f(x)$ **is equal to**

KCET 2021

Options:

A. -1

B. 1

C. 0

D. 3

Answer: A

Solution:

If $f(x)=\begin{vmatrix}\cos x & 1 & 0\\ 0 & 2\cos x & 3\\ 0 & 1 & 2\cos x\end{vmatrix}$

Expand along C_1,

$$\begin{aligned}&=\cos x\left(4\cos^2 x-3\right)\\&=4\cos^3 x-3\cos x\\&=\cos 3x\end{aligned}$$

$\therefore \lim_{x\to\pi}\cos 3x=\cos 3\pi=-1$

Question15

At $x=1$, the function

$$f(x)=\begin{cases}x^3-1, & 1<x<\infty\\ x-1, & -\infty<x\leq 1\end{cases} \text{ is}$$

KCET 2021

Options:

A. continuous and differentiable.

B. continuous and non-differentiable.

C. discontinuous and differentiable.

D. discontinuous and non-differentiable.

Answer: B

Solution:

$$f(x)=\begin{cases}x^3-1, & 1<x<\infty\\ x-1, & -\infty<x\le 1\end{cases}$$

We have to check the continuity at $x=1$.

$$\text{RHL} \Rightarrow \lim_{x\to1^+} f(x)=\lim_{x\to1^+}\left(x^3-1\right)=1-1=0$$

$$\text{LHL} \Rightarrow \lim_{x\to1^-} f(x)=\lim_{x\to1^-}(x-1)=1-1=0$$

$$f(1)=1-1=0$$

Thus the function is continuous at $x=1$.

$$f'(x)=\begin{cases}3x^2, & 1<x<\infty\\ 1, & -\infty<x\le 1\end{cases}$$

Now, check the differentiability at $x=1$.

LHD at $x=1 \Rightarrow 1$

RHD at $x=1 \Rightarrow 3(1)^2=3$

As, LHD $\neq$ RHD, function is not differentiable.

Question16

The right hand and left hand limit of the function are respectively.

$$f(x)=\begin{cases}\frac{e^{1/x}-1}{e^{1/x}+1}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$$

KCET 2020

Options:

A. 1 and 1

B. 1 and -1

C. -1 and -1

D. -1 and 1

Answer: B

Solution:

We have, $f(x)=\begin{cases}\frac{e^{1/x}-1}{e^{1/x}+1}, & \text{if } x\neq 0\\ 0, & x=0\end{cases}$

$$\begin{aligned}\text{Right hand limit } &= \lim_{x\to 0^+} f(x)=\lim_{h\to 0} f(0+h)\\ &=\lim_{h\to 0} f(h)=\lim_{h\to 0}\frac{e^{1/h}-1}{e^{1/h}+1}\\ &=\lim_{h\to 0}\frac{1-e^{-1/h}}{1+e^{-1/h}}\\ &=\frac{1-0}{1+0}=1 \quad \left(\because \lim_{h\to 0} e^{-1/h}=0\right)\\ \text{Left hand limit } &= \lim_{x\to 0^-} f(x)=\lim_{h\to 0} f(0-h)\\ &=\lim_{h\to 0} f(-h)\end{aligned}$$

$$=\lim_{h\to 0}\frac{e^{-1/h}-1}{e^{-1/h}+1}$$

$$=\lim_{h\to 0}\left(\frac{\frac{1}{e^{1/h}}-1}{\frac{1}{e^{1/h}}+1}\right)=\frac{0-1}{0+1}=-1$$

Question17

$\lim_{x\to 0}\left(\frac{\tan x}{\sqrt{2x+4}-2}\right)$ is equal to

KCET 2020

Options:

A. 2

B. 3

C. 4

D. 6

Answer: A

Solution:

We have,

$$\lim_{x\to 0}\left(\frac{\tan x}{\sqrt{2x+4}-2}\right)$$

$$=\lim_{x\to 0}\frac{(\tan x)(\sqrt{2x+4}+2)}{(2x+4)-4}$$
$$=\lim_{x\to 0}\frac{\tan x(\sqrt{2x+4}+2)}{2x}$$
$$=\frac{1}{2}\times(\sqrt{4}+2)=\frac{1}{2}(2+2)=2$$

Question18

If $f(x)=\begin{cases}\frac{1-\cos Kx}{x\sin x}, & \text{if } x\neq 0\\ \frac{1}{2}, & \text{if } x=0\end{cases}$ is continuous at $x=0$, then the value of K is

KCET 2020

Options:

A. $\pm\frac{1}{2}$

B. 0

C. ± 2

D. ± 1

Answer: D

Solution:

We have,

$$f(x)=\begin{cases}\frac{1-\cos kx}{x\sin x} & ,x\neq 0\\ \frac{1}{2} & ,x=0\end{cases}$$

$f(x)$ is continuous at $x=0$

$$\therefore \quad \lim_{x \to 0} \frac{1-\cos kx}{x \sin x} = \frac{1}{2}$$

$$\Rightarrow \quad \lim_{x \to 0} \frac{2\sin^2 \frac{k}{2}x}{x \sin x} = \frac{1}{2}$$

$$\Rightarrow \lim_{x \to 0} 2\left(\frac{\sin \frac{kx}{2}}{\frac{kx}{2}}\right)^2 \times \lim_{x \to 0} \frac{1}{\frac{\sin x}{x}} \times \frac{k^2}{4} = \frac{1}{2}$$

$$\Rightarrow \quad 2 \times \frac{k^2}{4} = \frac{1}{2}$$

$$\Rightarrow \quad k^2 = 1$$

$$k = \pm 1$$

Question19

If $f(x) = \begin{cases} \frac{\sin 3x}{e^{2x}-1}; & x \neq 0 \\ k-2; & x = 0 \end{cases}$ is continuous at $x = 0$, then $k =$

KCET 2019

Options:

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. $\frac{2}{3}$

D. $\frac{9}{5}$

Answer: A

Solution:

We have, $f(x) = \begin{cases} \frac{\sin 3x}{e^{2x}-1}; & x \neq 0 \\ k-2; & x = 0 \end{cases}$ is continuous at $x = 0$

$$\therefore \lim_{x\to 0} f(x) = f(0) \Rightarrow \lim_{x\to 0} \frac{\sin 3x}{e^{2x}-1} = k-2$$

$$\Rightarrow \lim_{x\to 0} \frac{\cos 3x(3)}{e^{2x}(2)} = k-2(\div \text{ form }) \Rightarrow \frac{3}{2} = k-2$$

$$\Rightarrow \quad k = \frac{3}{2} + 2 = \frac{7}{2}$$

Thus any option is not matching

Question20

$$\sum_{r=1}^{n}(2r-1) = x \text{ then, } \lim_{n\to\infty}\left[\frac{1^3}{x^2}+\frac{2^3}{x^2}+\frac{3^3}{x^2}+\ldots+\frac{n^3}{x^2}\right] =$$

KCET 2019

Options:

A. 1

B. $\frac{1}{2}$

C. 4

D. $\frac{1}{4}$

Answer: D

Solution:

We have, $\sum_{r=1}^{n}(2r-1) = x$

$\Rightarrow x =$ sum of first n odd natural numbers $\Rightarrow x = n^2$

Now, $\lim_{n\to\infty}\left[\frac{1^3+2^3+3^3+\ldots+n^3}{x^2}\right]$

$$\Rightarrow \lim_{x\to\infty}\left[\frac{\left(\frac{n(n+1)}{2}\right)^2}{(n^2)^2}\right] \Rightarrow \lim_{n\to\infty}\left[\frac{n^2(n+1)^2}{4n^4}\right]$$

$$\Rightarrow \lim_{n\to\infty}\left[\frac{(n+1)^2}{4\cdot n^2}\right] = \lim_{n\to\infty}\frac{1}{4}\left[\left(1+\frac{1}{n}\right)^2\right] = \frac{1}{4}(1+0)^2 = \frac{1}{4}$$

Question21

Rolle's theorem is not applicable in which one of the following cases?

KCET 2019

Options:

A. $f(x)=|x|$ in $[-2,2]$

B. $f(x)=x^2-4x+5$ in $[1,3]$

C. $f(x)=[x]$ in $[25,27]$

D. $f(x)=x^2-x$ in $[0,1]$

Answer: A

Solution:

Key Idea Rolle's theorem applicable when $f(x)$ be a real value function defined in $[a,b]$ such that (i) f is continuous is $[a,b]$ (ii) f is differentiable in (a,b) (iii) $f(a)=f(b)$ then there exists $c\in(a,b)$ such that $f'(c)=0$. We know that $f(x)=|x|$ is not differentiable at $x=0$.

Question22

The value of $\lim_{x\to 0}\frac{[x]}{x}$ is :

KCET 2018

Options:

A. 1

B. -1

C. 0

D. Does not exists

Answer: D

Solution:

We have,

$$\lim_{x \to 0} \frac{[x]}{x}$$

$$\text{LHL} = \lim_{x \to 0^-} \frac{[x]}{x} = \lim_{x \to 0-h} \frac{[0-h]}{0-h}$$

$$\Rightarrow \lim_{h \to 0} \frac{-1}{-h} = \infty$$

$$\text{RHL} = \lim_{x \to 0^+} \frac{[x]}{x} = \lim_{x \to 0+h} \frac{[0+h]}{h}$$

$$\Rightarrow \lim_{h \to 0} \frac{0}{h} = 0$$

$$\text{LHL} \neq \text{RHL}$$

Hence, limit does not exists.

Question23

If $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1} & \text{if } 0 \leq x \leq 1 \end{cases}$

is continuous at $x = 0$, then the value of k is

KCET 2018

Options:

A. $k = 1$

B. $k = -1$

C. $k = 0$

D. $k = 2$

Answer: B

Solution:

To ensure the function $f(x)$ is continuous at $x = 0$, the left-hand limit as x approaches zero from the negative side should equal the function's value at $x = 0$.

We have the function defined as:

$$f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1} & \text{if } 0 \leq x \leq 1 \end{cases}$$

Since we want the function to be continuous at $x = 0$, we require:

$$\lim_{x\to 0^-} f(x) = f(0)$$

For $x \to 0^-$, we look at:

$$\lim_{x\to 0^-} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}$$

Using the identity $(a-b)(a+b) = a^2 - b^2$, we can simplify:

$$\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} \cdot \frac{\sqrt{1+kx}+\sqrt{1-kx}}{\sqrt{1+kx}+\sqrt{1-kx}} = \frac{(1+kx)-(1-kx)}{x(\sqrt{1+kx}+\sqrt{1-kx})}$$

This simplifies further to:

$$\frac{2kx}{x(\sqrt{1+kx}+\sqrt{1-kx})} = \frac{2k}{\sqrt{1+kx}+\sqrt{1-kx}}$$

As $x \to 0^-$, the expression simplifies to:

$$\frac{2k}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2k}{2} = k$$

The function $f(x)$ at $x = 0$ is expressed by the right-side function evaluated as $x \to 0^+$:

$$f(0) = \frac{2(0)+1}{0-1} = -1$$

Thus, equating the limits:

$$k = -1$$

Hence, for the function $f(x)$ to be continuous at $x = 0$, the value of k must be -1.

Question24

If $f(x) = \begin{cases} \frac{\log_e x}{x-1} & ; x \neq 1 \\ k & ; x = 1 \end{cases}$

is continuous at $x = 1$, then the value of k is

KCET 2018

Options:

A. e

B. 1

C. -1

D. 0

Answer: B

Solution:

We have,

$$f(x)=\begin{cases}\frac{\log_e x}{x-1}; & x\neq 1\\ k; & x=1\end{cases}$$

Since this function is continuous at $x=1$

$\therefore \quad \lim_{x\to 1} f(x)=k$

$\Rightarrow \quad \lim_{x\to 1} \frac{\log_e \dot{x}}{x-1}=k$

Apply L'Hospital's rule,

$$\lim_{x\to 1}\frac{1}{x}=k$$
$$1=k$$
$$k=1$$

Question25

The value of $\lim_{\theta\to 0}\frac{1-\cos 4\theta}{1-\cos 6\theta}$ *is*

KCET 2017

Options:

A. $9/4$

B. $9/3$

C. $4/9$

D. $3/4$

Answer: C

Solution:

$$\lim_{\theta\to 0}\frac{1-\cos 4\theta}{1-\cos 6\theta}$$
$$=\lim_{\theta\to 0}\frac{2\sin^2 2\theta}{2\sin^2 3\theta}$$
$$=\lim_{\theta\to 0}\frac{\sin^2 2\theta}{4\theta^2}\cdot\frac{1}{\frac{\sin^2 3\theta}{9\theta^2}}\cdot\frac{4\theta^2}{9\theta^2}$$
$$=\lim_{\theta\to 0}\left(\frac{\sin 2\theta}{2\theta}\right)^2\cdot\frac{1}{\left(\frac{\sin 3\theta}{3\theta}\right)^2}\cdot\frac{4}{9}\quad\left[\because\lim_{x\to 0}\frac{\sin x}{x}=1\right]$$
$$=\tfrac{4}{9}$$

Question26

If $f(x)=\begin{cases}kx^2 & \text{if} \quad x\le 2\\ 3 & \text{if} \quad x>2\end{cases}$ is continuous at $x=2$, then the value of k is

KCET 2017

Options:

A. $3/4$

B. 4

C. $4/3$

D. 3

Answer: A

Solution:

We have,

$$f(x)=\begin{cases} kx^2, x \leq 2 \\ 3, x>2 \end{cases}$$

Since, $f(x)$ is continuous at $x=2$

$$\begin{aligned}
\therefore \quad & \text{LHL (at } x=2) = \text{ RHL (at } x=2) \\
\Rightarrow \quad & \lim_{x \rightarrow 2^-} kx^2 = \lim_{x \rightarrow 2^+} 3 \\
\Rightarrow \quad & \lim_{h \rightarrow 0} k(2-h)^2 = 3 \\
\Rightarrow \quad & k(2-0)^2 = 3 \\
\Rightarrow \quad & 4k = 3 \\
\Rightarrow \quad & k = \frac{3}{4}
\end{aligned}$$
